Course: PHYS 516
Term: Fall 2015
Room: Herman Brown 453
Class: TTh, 4:00-5:15

INSTRUCTOR CONTACT INFORMATION

Instructor: Matthew G Baring
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Office Hours: TTh 1:00-2:30pm

COURSE OBJECTIVES AND LEARNING OUTCOMES

Mathematical Methods is taught at the graduate level. It gathers together many of the concepts and techniques that are discovered in specialized undergraduate mathematics courses, highlighting their application to physics pedagogy so as to facilitate learning throughout Rice's Physics and Astronomy graduate curriculum. It is therefore to a large extent a survey course, and complete mathematical rigor and depth is not possible in such a condensed format. Yet the material presented provides an excellent starting point for those interested in delving deeper into particular subfields. For graduates, it serves as a grounding for problems they might tackle in their research work. For any undergraduates taking this course, it serves a fast-track in providing the mathematical foundation for future graduate studies in physics or astronomy.

Objectives: The goal is to provide students with a basic understanding of many of the important elements of mathematics, developing a working knowledge of standard mathematical techniques that are useful to physicists and astronomers, especially for problems involving differential equations (ordinary and partial), integrals and their transforms, special functions, complex analysis, series and matrices. While much of the course is analytic in character by necessity, to maintain contact with contemporaneous computational methods as research tools, there is also a significant numerical component to the course. This will address algorithm development and also provide practical opportunities at numerics for contained problems in applied mathematics. Topics covered include interpolation and curve fitting, iterative solution of ordinary differential equations, numerical integration and quadrature, matrix decomposition methods, and finite difference approaches to partial differential equations.

Students will learn how particular techniques can be applied to different physics disciplines. They will learn how to assemble various of elements of mathematics from disparate areas, and integrate them to cohesively to attack a particular problem. Students will be taught to think logically and critically about what are reasonable assumptions in mathematical developments and what are not, and how to evaluate and check derived results. They will also be taught how to think "laterally" in the sense of connecting seemingly unrelated techniques in broaching a given task. This will apply both to analysis methods and also in developing computational skils.

Learning Outcomes: By completing the course, students will be much better equipped to use mathematical tools in their research, thereby enhancing the Rice graduate study experience. They will be in a position to assess posed problems, suggest algorithms for approaching analytic and computational tasks, test their ideas, and develop efficient strategies for reducing a problem to a tractable form, or an endpoint that will facilitate numerical evaluation. They will receive training to communicate their ideas, hypotheses and understanding of select issues to an ensemble of their peers. This is the essence of the research process: think critically, probe, discover, revise one's perspective and tell the broader community of the results and the path taken to get there, and where to go next. This is invaluable training for an array of professions and potential careers down the line, including industry, business and academia.

REQUIRED TEXTS AND MATERIALS

Mathematical Methods for Physicists, by George Arfken and Hans Weber (Elsevier, Amsterdam)

We will structure the course around the Sixth Edition (2005) of this book: it is an essential supplement to the lectures. The 7th edition of this book, *Mathematical Methods for Physicists*, by Arfken, Weber & Harris (Elsevier, 2012), can also be used. Notes for course material not contained therein will be provided, for example from Mathews and Walker, and Garcia, listed below.

SUPPLEMENTARY SUPPORTING TEXTS

Mathematical Methods of Physics, by Jon Mathews & Robert L. Walker. (Addison-Wesley, New York)
[Fragments of a more formal nature will come from this text throughout the course.]

Numerical Methods for Physics, by Alejandro L. Garcia (Prentice Hall, New Jersey)
[Numerical elements on differential equations will be drawn from this text.]

EXAMS AND PAPERS

The course assessment will consist of approximately eight to ten problem sheets, cumulatively constituting 65% of the total grade, one ***closed-book*** **mid-term exam** during the semester that constitutes 15% of the grade, and an ***open-book, open notes*** **take-home final exam** at the end of the semester, constituting the remaining 20% of the assessment.

GRADE POLICIES

All parts of the assessment will be graded on a curve, determined commensurately with the overall performance of past students who have taken this course at Rice. This means that present students will not only be measured relative to their peers, but also relative to the long-term body of high-caliber Rice students who have enjoyed the experience of this course.

Late homeworks will automatically receive a 5% reduction in credit, unless an extension has been negotiated with Prof. Baring. Homeworks that are 4-7 days or more overdue will be reduced by 30% in total credit. Beyond that timeframe, late homeworks will not be graded and score zero. This policy is because it is (i) not fair to other students to have the return of their homeworks in a timeframe that is delayed by inadvertent tardiness by any student, and (ii) it is not fair to impose logistical constraints on Prof. Baring in terms of grading.

Extensions of homework deadlines must be negotiated with Prof. Baring prior to the original deadline, with the student defining good cause for the extension. The negotiated deadline will substitute for the original one in terms of the aforementioned late penalties.

The final exam must be submitted prior to the University-mandated deadline of 5pm on Wednesday, 16th December, the end of Fall Semester, 2015.

Exceptions to these late policies can occur for extenuating circumstances such as student illness, family illness or emergency. In such cases, it is the student's responsibility to let Prof. Baring know (ahead of time, if possible) what is going on so that he is not "in the dark." The student will need to (retroactively) document the circumstances.

CLASS ATTENDANCE

The purpose of the lectures is to impart knowledge distilled to its essentials on the subject matter of the course and in a manner more efficient than is afforded by merely reading textbooks and browsing Web sites; these important out-of-class learning paths are intended to supplement, not replace lectures. A central ingredient of this classroom forum is leveraging the extensive research experience and science connections of the Lecturer, *and this is best done by attending lectures*.

The small average class size underpins an exceptional learning experience that sets Rice apart from many of its peer institutions. **Students should take advantage of this opportunity by habitually attending classes**; their learning curve will be enhanced by such dedication.

ABSENCE POLICIES

Infrequent absences are not a problem. If a student is noted to be absent for an extended period of time, or frequently, the student must communicate with Prof. Baring the reasons of the absence(s). Such cases normally will degrade the efficiency of learning for the student. Again, if there are extenuating circumstances such as student illness or family illness or emergency, accommodations will be made, and Prof. Baring should be informed. Otherwise, concerning any absence, it is the student's responsibility to acquire the pertinent notes/materials to guide their study accordingly. Prof. Baring is under no obligation to provide copies of notes to students who do not attend a given lecture, nor to "re-lecture" such material during office hours.

IN-CLASSROOM TECHNOLOGIES

The learning environment for individuals and the entire class is optimized if it is not disrupted by cell phone activity. Use of cell phones to text or via another mode is distracting to the lecturer and shows inattention on the part of the perpetrator. Moreover it is rude to the lecturer, who invests considerable time in preparing lectures to facilitate the learning of the entire class, and to other students when a distracted moment arises. Dr. Baring **prohibits the use of cell phones in the lecture room**; if a student cannot wait until the conclusion of class to send a text or make a call, he or she should quickly excuse themselves, leave the room, and return only when finished and ready to concentrate on the lecture. Use of laptop computers to take notes is not intrusive and is acceptable to Dr. Baring; use of them to perform telecommunication functions such as Skyping and email is similarly not permitted. **Violations of these rules will lead to the student being asked to leave the classroom for the remainder of the lecture.**

RICE HONOR CODE

In this course, all students will be held to the standards of the Rice Honor Code, a code that you pledged to honor when you matriculated at this institution. If you are unfamiliar with the details of this code and how it is administered, you should consult the Honor System Handbook at http://honor.rice.edu/honor-system-handbook/. This handbook outlines the University's expectations for the integrity of your academic work, the procedures for resolving alleged violations of those expectations, and the rights and responsibilities of students and faculty members throughout the process.

The mid-term and final (take-home) exam questions are not to be discussed at all with other students, faculty or graders, and are subject to the provisions of the Rice Honor Code. Please verify this by **writing the word pledge and your signature on each exam**. Questions specifically about exams should be directed only to Prof. Baring.

DISABILITY SUPPORT SERVICES

If you have a documented disability or other condition that may affect academic performance you should: 1) make sure this documentation is on file with Disability Support Services (Allen Center, Room 111 / adarice@rice.edu / x5841) to determine the accommodations you need; and 2) talk with Prof. Baring to discuss your accommodation needs *during the first two weeks of class*.

Any letter from DSS to the instructor requesting accommodations for the student should be delivered in the first three weeks of semester, so that Prof. Baring can plan accordingly.

SYLLABUS

The detailed syllabus below gives the layout of the course material. For further information, such as scheduling, pointers to related chapters in the Required Text, etc., see the PHYS 516 course web pages at http://spacibm.rice.edu/%7Ebaring/phys516/phys516_syllabus.html/ .

Complex Analysis

Uses of Complex Variables
Functions of a Complex Variable
    Complex Functions of a Complex Variable
Differentiation
    The Cauchy-Riemann Relations
Integration
    The Cauchy Integral Theorem
    The Cauchy Integral Formula
Taylor and Laurent Series
The Theorem of Residues

Interpolation, Fitting and Root Solving

Lagrange Interpolation
Cubic Splines
Least Squares Fitting
Padé Approximations
Root Solving
    Numerical Bisection

Ordinary Differential Equations

Linear, First Order Ordinary Differential Equations
Linear ODEs with Constant Coefficients
Linear ODEs with Non-Constant Coefficients
Power Series Solutions: Frobenius' Method
WKB Approximation
Numerical Solutions: Euler's Method
Runge-Kutta Technique

Series and Summation

Common Examples of Infinite Series
Convergence
    D'Alembert's and Cauchy's Ratio Test
    The Cauchy-Maclaurin Integral Test
    Gauss' Refined Ratio Test
Improvement of Convergence
    Kummer's Technique
    Rational Approximation
    Euler's Transformation

Integration

- Special Devices for Particular Cases
  - The Power of Differentiation
  - Series Expansions
- Contour Integration
- Asymptotic Expansions
  - Laplace's Method of Steepest Descents
- Numerical Integration
  - Newton-Cotes Formulae
  - Gaussian Quadrature
  - Romberg's Method

Fourier Series and Transforms

- Connecting Fourier Series
  - Complex Form for Fourier Series
- The Fourier Transform
  - Parseval's Relation and Uncertainty Principal

Integral Transforms and Dispersion Relations

- Various Integral Transforms
  - Generic Properties
  - Laplace Transform Applications
- Dispersion Relations
  - Hilbert Transforms
  - Optical Dispersion

Vectors and Matrices

- Linear Vector Spaces
- Linear Operators and Matrices
  - Coordinate Transformations
  - Normalization of Vectors
- Eigenvalue Problems
  - Gram-Schmidt Orthogonalization
- Hermitian Matrices and Diagonalization
- Matrix Inversion
  - Gauss-Jordan Elimination
  - LU Decomposition

Special Functions I

- The Gamma Function
  - Definitions and Simple Properties
  - Polygamma Functions
  - Stirling's Series
- Functions Related to the Gamma Function
  - Beta and Incomplete Gamma Functions
  - Riemann Zeta Function

Special Functions II: Bessel Functions

- Bessel Functions J_nu and N_nu
  - Generating Function and Recurrence Relations
  - An Integral Representation
  - Neumann Functions and Wronskians
- Modified Bessel Functions I_nu and K_nu
  - Integral Representations and Asymptotic Series

Special Functions III: Orthogonal Polynomials

- The Legendre Polynomials P_n
  - Legendre Functions of the 2nd Kind Q_n
  - Associated Legendre Functions
- Hermite Polynomials H_n
- Laguerre Functions
- Hypergeometric Functions

Partial Differential Equations

- Global Characterization of PDEs
- Solution of Linear Second Order PDEs
  - Separation of Variables
  - Transform Techniques
- Numerical Solutions of PDEs
  - Initial and Boundary Value Problems
  - Method of Images
  - FTCS Schemes
  - Relaxation Methods

Eigenfunctions

- Eigenfunctions and Eigenvalues for Linear PDEs
- Homogeneous Problems: Green's Functions
  - Green's Functions for Initial Conditions

Integral Equations

- Overview
- Transform Techniques
- Neumann Series Solutions
- Hilbert-Schmidt Theory

SYLLABUS CHANGE POLICY

This syllabus is a general guide for the course and is subject to change without advanced notice.